4.5 Negative Exponents and Reciprocals

LESSON FOCUS	Relate negative exponents to reciprocals.

$\frac{3}{4} \div \frac{4}{3}$

Copy & Complete the following:

Ex: $\frac{2^4}{2^2} = 2^2$	$\frac{2\cdot2\cdot2\cdot2}{2\cdot2} = 2^2 = 4$	$\frac{16}{4} = 4$
$\frac{2^4}{2^3} = 2^1$	$\frac{2\cdot2\cdot2\cdot2}{2\cdot2\cdot2} = 2$	$\frac{16}{8} = 2$
$\frac{2^4}{2^4} = 2^0$	$\frac{2\cdot2\cdot2\cdot2}{2\cdot2\cdot2\cdot2} = 1$	$\frac{16}{16} = 1$
$\frac{2^4}{2^5} = 2^{-1}$	$\frac{2\cdot2\cdot2\cdot2}{2\cdot2\cdot2\cdot2\cdot2} = \frac{1}{2}$	$\frac{16}{32} = \frac{1}{2}$

$2^0 = 1$

$2^{-1} = \frac{1}{2}$

$\frac{2^4}{2^6} = 2^{-2} = \frac{1}{2^2}$

$5^{-3} = \frac{1}{5^3} = \frac{1}{125}$

$\frac{3}{4}$ $\frac{4}{3}$

Copy & Complete the following:

Ex: $\frac{2^4}{2^2} = 2^2$	$\frac{2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2} = 2^2$	$\frac{16}{4} = 4$
$\frac{2^4}{2^3} = 2^1$	$\frac{2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2} = 2^1$	$\frac{16}{8} = 2$
$\frac{2^4}{2^4} = 2^0$	$\frac{2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2 \cdot 2} = 2^0$	$\frac{16}{16} = 1$
$\frac{2^4}{2^5} = 2^{-1}$	$\frac{2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} = 2^{-1}$	$\frac{16}{32} = \frac{1}{2}$

$(-2)^{-3} = \frac{1}{(-2)^3}$

$\frac{1}{(-3)^{-4}} = (-3)^4$

$\frac{1}{4^{-3}} = 4^3$

$$9^{-\frac{3}{2}} = \frac{1}{9^{3/2}} = \frac{1}{\left(\sqrt{9}\right)^3}$$

$$= \frac{1}{3^3} = \frac{1}{27}$$

Ex:

$$27^{-\frac{2}{3}} = \frac{1}{27^{2/3}} = \frac{1}{\left(\sqrt[3]{27}\right)^2} = \frac{1}{(3)^2} = \frac{1}{9}$$

Negative Exponent Law

Fractional Exponents

$$100^{-\frac{3}{2}} = \frac{1}{100^{3/2}} = \frac{1}{\left(\sqrt{100}\right)^3} = \frac{1}{10^3}$$

$$= \frac{1}{1000}$$

$$= 0.001$$

Please try all of the following:

Q Identify and explain the errors in the following:

1st set (whole number exponents):

#1 a) $4^3 + 4^2 = 4^5$ b) $\frac{x^6}{x^3} = x^2$ c) $(10^2)^5 = 10^7$ d) $\left(\frac{1}{4}\right)^2 = \frac{1}{8}$

e) $(x - y)^3 = 3x - 3y$ f) $3^5 \times 3^2 = 3^{10}$ g) $5^3 \div 5^4 = \frac{3}{4}$

2nd set (integral exponents):

#2 a) $a^4 \cdot a^{-2} = a^{-8}$ b) $b^{-10} \div b^5 = b^{-5}$ c) $(c^{-3})^2 = c^{-1}$ d) $\left(\frac{2}{3}\right) - 2 = \frac{-4}{-6}$

3rd set (rational exponents):

#3 a) $2^{\frac{1}{2}} \cdot 2^{\frac{1}{2}} = 2^{\frac{1}{4}}$ b) $3^{\frac{3}{4}} \div 3^{\frac{1}{4}} = 3^3$ c) $\left(4^{\frac{2}{5}}\right)^2 = 4^{\frac{2}{25}}$ d) $\left(5^{\frac{1}{2}}\right)^2 = 5^{2^{\frac{1}{2}}}$

Q Fill in the blanks:

#4 a) $5^{-2} = \frac{\square}{\square}$ b) $6^{\square} = \frac{1}{6^2}$ c) $\square^{-6} = \frac{1}{10^6}$ d) $4^{-x} = \frac{1}{\square}$

Q Solve the following, by substituting the values given:

#5 a) $5x^4 + 6xy \quad if \quad x = 2, \; y = 3$

b) $(2x)^2 \; if \; x = 4$

c) $(t + s)^{-3} \; if \; t = 2, \; s = 4$

Show your work

Activity

TRY THIS

Work with a partner.

You will need grid paper and scissors.

A. Cut out a 16 by 16 grid. Determine the area of the grid in square units and as a power of 2. Record your results in a table like this:

Cut	Area (units^2)	Area as a Power of 2
Start	256	
1		
2		
3		

B. Cut the grid in half and discard one piece.
In the table, record the area of the remaining piece in square units and as a power of 2.

C. Repeat Step B until the paper cannot be cut further.

4.5 Negative Exponents and Reciprocals

Two numbers with a product of 1 are reciprocals.

Since $4 \cdot \frac{1}{4} = 1$, the numbers 4 and $\frac{1}{4}$ are reciprocals.

Similarly, $\frac{2}{3} \cdot \frac{3}{2} = 1$, so the numbers $\frac{2}{3}$ and $\frac{3}{2}$ are also reciprocals.

We define powers with negative exponents so that previously developed properties such as $a^m \cdot a^n = a^{m+n}$ and $a^0 = 1$ still apply.

Apply these properties.

$$5^{-2} \cdot 5^2 = 5^{-2+2}$$
$$= 5^0$$
$$= 1$$

Since the product of 5^{-2} and 5^2 is 1, 5^{-2} and 5^2 are reciprocals.

So, $5^{-2} = ?$ and $\frac{1}{5^{-2}} = ?$

That is, $5^{-2} = ?$

This suggests the following definition for powers with negative exponents.

4.5 Negative Exponents and Reciprocals

NEGATIVE EXPONENTS

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x^{-n} is defined to be the reciprocal of x^n

That is $x^{-n} = \frac{1}{x^n}$, $(x \neq 0)$

~~Exponent Law #5~~

NEGATIVE EXPONENTS

$3^{-1} = \frac{1}{3}$

Remember that a negative exponent does not mean a negative number but the *reciprocal number.*

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$4^{-3} = \frac{1}{4^3}$

WHY IS THIS TRUE?

- WATCH THIS PATTERN
- $10^3 = 1000$
- $10^2 = 100$
- $10^1 = 10$
- $10^0 = 1$
- $10^{-1} = 0.1$ or $\frac{1}{10}$
- $10^{-2} = 0.01$ or $\frac{1}{100}$

Zero Exponent –

Negative Exponent –

EXAMPLE

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Long Version

$$\frac{4^{-5}}{4^{-3}}$$

4^5 4^2

Short Version

$$\frac{4^{-5}}{4^{-3}}$$

d

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Powers with Negative Exponents

When x is any non-zero number and n is a rational number, x^{-n} is the reciprocal of x^n.

That is, $x^{-n} = \frac{1}{x^n}$ and $\frac{1}{x^{-n}} = x^n$, $x \neq 0$

4.5 Negative Exponents and Reciprocals

Example 1 Evaluating Powers with Negative Integer Exponents

Evaluate each power.

a) 3^{-2} b) $\left(-\frac{3}{4}\right)^{-3}$ c) 0.3^{-4}

SOLUTION

CHECK YOUR UNDERSTANDING

Example 1 Evaluating Powers with Negative Integer Exponents

Evaluate each power.

a) 3^{-2} b) $\left(-\frac{3}{4}\right)^{-3}$ c) 0.3^{-4}

SOLUTION

a) $3^{-2} = \frac{1}{3^2}$

$= \frac{1}{9}$

b) $\left(-\frac{3}{4}\right)^{-3} = \left(-\frac{4}{3}\right)^{3}$

$= -\frac{64}{27}$

(Solution continues.)

4.5 Negative Exponents and Reciprocals

Example 1 Evaluating Powers with Negative Integer Exponents

c) 0.3^{-4}

Use a calculator.

$0.3^{-4} = 123.4567\ldots$

CHECK YOUR UNDERSTANDING